Instabilities in a bimode CO_2 laser with a saturable absorber

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We report on the observation of mode hopping in a bimode CO_2 laser with a saturable absorber. When the control parameters are changed, the output intensity of this laser shows a succession of periodic regimes that may become completely erratic. Each of these regimes may be explained considering the role played alternatively by two unstable fixed points corresponding to the steady state intensity of each mode. In a particular range of the parameter space, a description of multipeaked regimes is given using the Farey arithmetic, and it ressembles that observed in the Belousov–Zhabotinskii reaction. Transverse effects are observed that result from the nonlinear coupling of transverse cavity modes with both the active and passive media.

Multimode operation spontaneously occurs in many lasers, resulting in large amplitude fluctuations and instabilities due to the nonlinear coupling of the different modes through the amplifying medium, but the detailed study of the mechanisms leading to instabilities is rather involved. In the past few years, the rapid development of nonlinear dynamics has provided us with tools for a new insight into the behavior of these lasers. The simplest case of a multimode laser is the two-mode laser that has been largely investigated both experimentally and theoretically [1–3]. A more complete bibliography on two-mode behavior is given in ref. [4]. For instance, the bidirectional ring laser is an interesting case of a two-mode laser where two counterpropagating waves are interacting via the active medium. This interaction gives rise to effects such as bistability and mode hopping [5–9]. YAG lasers with an intracavity doubling crystal also exhibit instabilities due to the nonlinear coupling between two or several longitudinal modes [10]. Other types of two-mode behavior have also been investigated in the past few years, including polarization switching in a standing wave cavity [11] and oscillation of modes in extended media in presence of spatial inhomogeneity in the pumping rate [12]. More recently, Lugiato et al. studied the interaction between transverse modes in a laser [13] and Tredicce et al. showed experimental evidence of multimode induced complex spatial structure in a CO_2 laser [14].

The CO_2 laser containing a saturable absorber (LSA) has never been investigated experimentally in a multimode regime. Here, we report on the first observation of two-mode dynamics in a LSA. This system differs from the two-mode lasers previously studied since our monomode LSA is operated in a regime in which it displays spontaneous pulsations known as passive Q-switching, while the monomode operation of the bidirectional or YAG lasers is stable. The LSA is known to display monomode instabilities and Shil'nikov chaos and we have studied in simple cases how the LSA dynamics is affected when the laser goes from monomode to multimode operation. Most of these instabilities may be explained considering the role played alternatively by two unstable fixed points corresponding to the steady state intensity of each mode. In a previous paper, we have studied the response of a Q-switching LSA to a periodic external forcing field that modulates the absorber [15]. In this case the response of the modulated LSA depends on the interaction of an internal clock with an external one, while here, two internal clocks are interacting in an all optical system.

Our paper is organized as follows: we report on experiments that illustrate the role played by the two fixed points, then we analyze the conditions of coexistence of the two Q-switching modes that can possibly lock to each other and give rise to periodic output. A complete description of the periodic states is given using the Farey arithmetic, and we show that the behavior of the LSA resembles that observed in the Belousov–Zhabotinskii reaction [16]. Finally, the spatial dependence of the laser intensity resulting from the nonlinear coupling of transverse cavity modes with the intracavity media is discussed.

The experimental device is the same as that described in ref. [17]. It is composed of a CO_2+CH_3I LSA operating on the 10P32 line, with pressures in the absorber and amplifier of 30 mTorr and 25 Torr respectively. Both the grating and the output cavity coupler have been slightly tilted to favor the appearance of two transverse modes called hereunder mode 1 and mode 2, with only a partial overlapping of their spatial structure. Here, in the absence of absorber, the two modes coexist over a large range of the cavity detuning, and give rise to a beat signal on the detector. The observed behaviors do not depend critically on the tilt angle as long as two or more modes are above threshold. Measurements of the transverse profile of the modes indicate that the symmetry properties of cylindrical cavities are completely lost because of the mirror tilt. The values of both pump parameter and loss may be significantly different from mode 1 to mode 2. However, we have checked that similar dynamics occurs in another laser that saves the cylindrical symmetry and in which multimode operation is achieved by an intracavity optical device. This supports the fact that the dynamical effects presented here have a general character and that they are not qualitatively affected by the loss of the cylindrical symmetry.

The output power is monitored on a HgCdTe detector through a 0.5 mm diameter iris placed in front of it. A scanner associated with a 10× beam expander allows us to monitor the transverse structure of the laser fields. This detection is only sensitive to the total intensity (mode 1+mode 2). Heterodyne detection has also been used to separate mode 1 from mode 2. In this system, a stable free running waveguide CO_2 laser provides a frequency reference and the beat between the LSA and the reference laser is analyzed. This system allows us to measure the frequency spacing between the two modes that is typically 15 MHz and remains approximately constant with the cavity length.

The bimode operation of the CO_2 LSA gives rise to a complex scenario that may be explained on the basis of what is observed in monomode operation. Let us recall briefly the evolution of this single mode operation as a function of the cavity detuning. The cavity length is chosen as the control parameter, since it acts directly on the laser detuning. We start from a detuning value where the laser intensity is stable (cw operation). As the detuning is increased, the laser destabilizes through a Hopf bifurcation opening a Feigenbaum route to chaos. For larger detunings, the laser emerges temporarily from chaos to emit $P^{(n)}$ type signals well understood in the framework of the Shil'nikov dynamics theory. Similar scenarii have been observed with detunings of opposite signs. More details on the monomode behavior of the LSA are given in refs. [17–21].

Let us consider now the bimode operation of the LSA. As the intermode frequency spacing (15 MHz) is small compared with the cavity free spectral range (50 MHz), we have investigated the cavity length range where each of the two modes would be unstable when considered separately. When the two modes operate simultaneously, the position of zero detuning of each mode is more difficult to localize. So we found experimentally much easier to take the zero detuning reference at the point where the laser switches from single to bimode operation when increasing the cavity length. With this particular choice, the origin is not absolutely fixed as it depends strongly on the experimental parameters but it allows direct comparison of sequences of signals taken with the cavity length as a single control parameter. Depending on the experimental conditions, the width of the bimode cavity length range lies generally between 1 and 4 μm. In the bimode tuning range under investigation, if only mode 1 is allowed to oscillate, i.e. when mode 2 is artificially blocked up, the output signal evolves as a function of the cavity detuning from a cw regime to a $P^{(0)}$ one. At the same time, mode 2 would evolve from $P^{(0)}$ to cw. When the two modes are expected to coexist, they are coupled via both the active and passive molecular media so that their temporal evolutions are not independent.

Let us discuss the possible coupling mechanisms between the two modes. Strong coupling occurs when two modes interact with the same molecules inside the medium, because their frequency difference is small with respect to the homogeneous width. Two phenomena are responsible for a limitation of the strength of coupling. The first one is Doppler effect, since within the experimental conditions, Doppler broadening (≈ 60 MHz) is not negligible with respect to the homogeneous broadening (≈ 120 MHz) in the active medium and much larger than the broadening (≈ 0.5 MHz) in the passive medium. The second one arises from the limited spatial overlap of the transverse field patterns of the two modes. As a consequence from their different transverse extension, they do not interact strongly with the same molecules.

When the laser goes from monomode to multimode operation, different regimes may occur. Apart from the trivial permanent monomode *Q*-switching, i.e. on mode 1 or on mode 2 while the other mode is off, that appears when losses of the two modes are very different or in case of strong coupling, various regimes have been observed that will be discussed now.

Periodic quasi-discontinuous jumps from one mode to the other one, known as "mode hopping", have been obtained. Fig. 1a shows the periodic evolution of the total intensity versus time in such a regime. Then the laser switches from mode 1 to mode 2 and vice versa. During a period of the instability oscillation, the output intensity of one mode first grows exponentially from zero and then exhibits a large spike followed by several undulations similar to those emitted by the LSA operated in the monomode regime [17–21]. Then, the intensity of this mode decreases rapidly to zero and remains there while the second mode is oscillating. When starting oscillation, one mode kills the other mode present in the cavity, and it has been checked using the heterodyne detection, that the time of dual emission is in most cases negligible compared with the period of the signal ($\Delta T/T < 1/20$). In our bimode experimental situation the minimum of the global intensity never approaches zero except at low gain or at high absorber pressure. On the opposite, at low absorber pressures, it was shown that in monomode *Q*-switching regime, the LSA intensity gets closer to zero as the laser is operated further from the Hopf bifurcation by varying the cavity length [17].

The main characteristics of the trajectory of the system appear on a 2D phase space reconstruction. The reconstruction of this trajectory is performed from the signal of fig. 1a, using the time delay technique (fig. 1b). The dynamics of the system may be interpreted as follows: it appears to be mostly influenced by two unstable fixed points I_1 and I_2 that correspond to the steady state values of the intensities of the isolated modes. The trajectory is made of converging spirals in a plane containing I_2, that acts as a repeller in the other directions. Periodically, the trajectory leaves the vicinity of I_2 to visit a region far from the fixed points domain before being reinjected in the vicinity of a plane containing I_1. After some revolutions around I_1, it is then repelled far from I_1 before being reinjected near I_2. Dual emission only happens when the system is repelled far from the fixed points I_1 and I_2.

The number of revolutions around I_1 and I_2 varies as a function of various parameters such as the pressures in the intracavity media, the discharge current and the cavity length. We have studied the transition scenario from n to $n+1$ revolutions using the cavity length as a control parameter. Fig. 2 describes a part of a complete scenario in which the number of revolutions around I_2 increases while it decreases around I_1. The period of the oscillations around the fixed points is close to that of the ordinary type II *Q*-switching (see refs. [17–19]) in monomode operation. In fig. 2a, the lowest intensity mode 1 emits two undulations, then switches off while mode 2 switches on to emit three undulations. A small variation in the control parameter results in a sudden qualitative change of the system as it emits now signals with three and four undulations alternatively (fig. 2b). After a next small variation (fig. 2c), the signal recovers a period similar to the previous one with always four undulations for I_2. The evolution of the system with the cavity detuning may be summarized by the scheme

$$(2,3,2,3) \rightarrow (1,4,2,3) \rightarrow (2,4,2,4)$$

where the figures indicate successively the number of revolutions around the fixed points I_1 and I_2 as illustrated on fig. 2a, 2b, 2c. As these successive transitions occur on a very small part ($\approx 1/50$) of the cavity free spectral range (≈ 50 MHz) and because of frequency jitter, it has not been possible to search

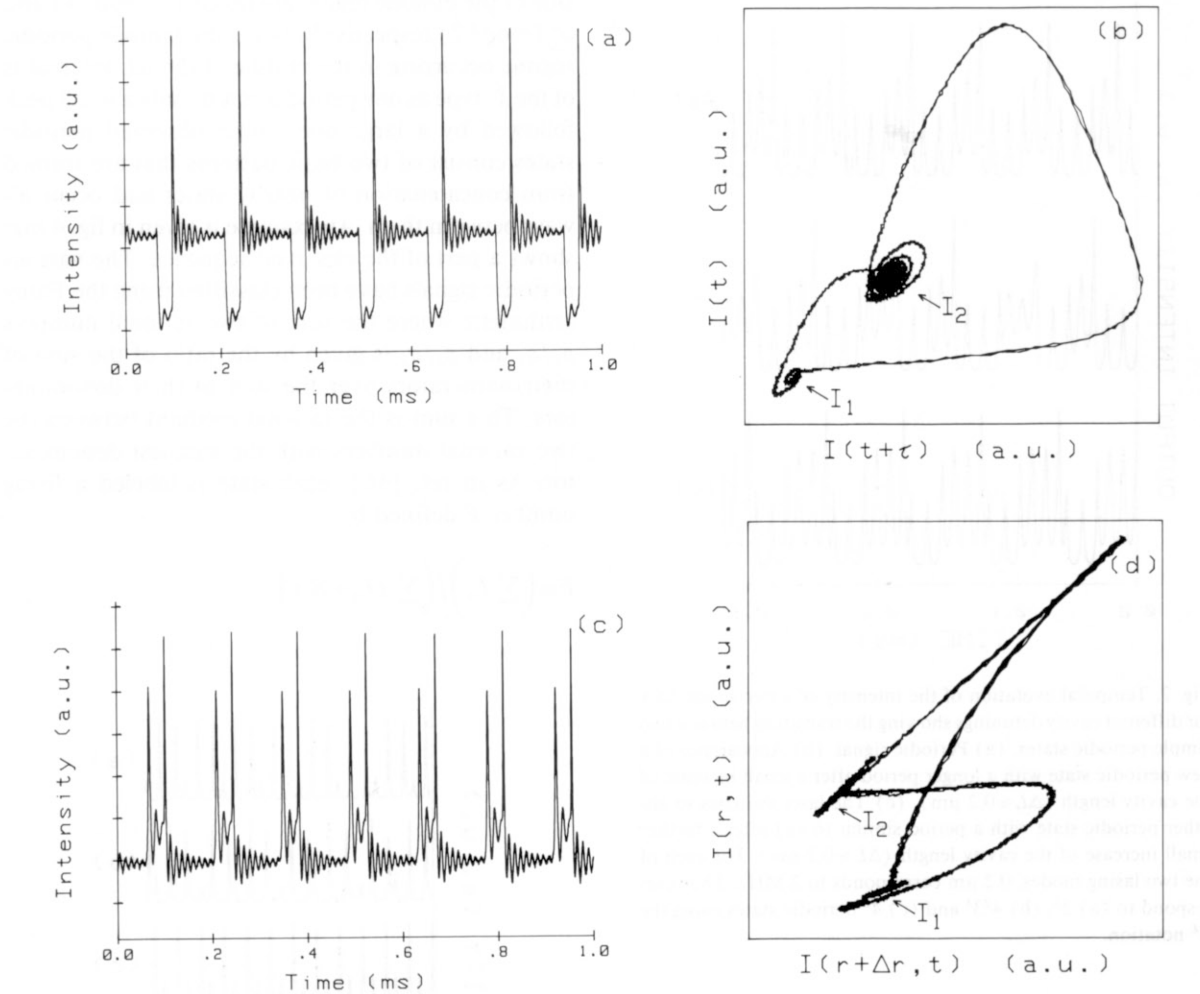

Fig. 1. (a), (c) Temporal evolution of the intensity of a two-mode LSA taken in two different points along a transverse direction of the output beam. (b) Phase space reconstruction $I(t)$, $I(t+\tau)$ of (a). (d) Phase space reconstruction $I(r,t)$, $I(r+\Delta r,t)$ from the data of (a) and (c).

systematically for other intermediate regimes that could possibly hold between those which have been presented above. However signals with a much longer period have been observed during a few cycles as it will be explained in more detail hereunder.

In order to obtain a better understanding of this particular behavior in a simpler case, another experiment has been performed in a region of the parameter space where the two transverse modes are Q-switching in a $P^{(0)}$ type regime that consists of pulses without fine structure. The $P^{(0)}$ signals obtained at each side of the ΔL tuning range ($\Delta L=4$ μm) where the laser is monomode, differ only by their amplitude and period (see fig. 3). We have checked on complex regimes recorded in a bimode operation that small peaks correspond to the response of one mode while large peaks are due to the other one. In the ΔL region under investigation, the bimode CO_2 LSA exhibits complex periodic or irregular oscillatory behaviors. As the cavity length is changed, the system scans successively various periodic states.

A compact description of these multipeaked periodic states is provided by the Farey arithmetic as

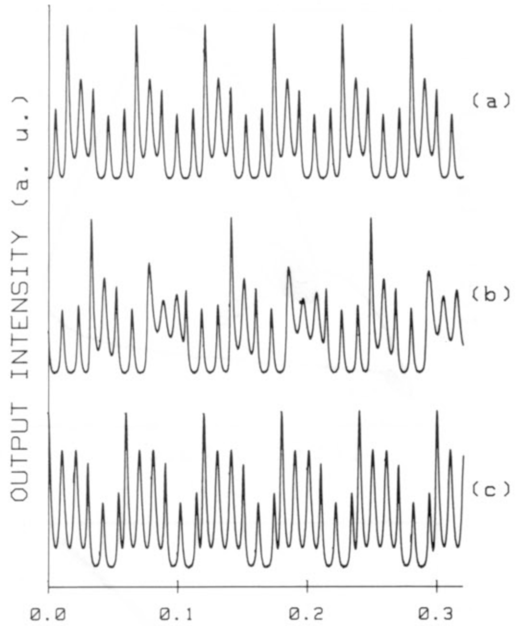

Fig. 2. Temporal evolution of the intensity of a two-mode LSA for different cavity detunings showing the transition between two simple periodic states. (a) Periodic signal. (b) Appearance of a new periodic state with a longer period after a small increase of the cavity length ($\Delta L \approx 0.2$ μm). (c) The laser switches to another periodic state with a period similar to (a) after a further small increase of the cavity length ($\Delta L \approx 0.2$ μm). For each of the two lasing modes, 0.2 μm corresponds to 2 MHz. They correspond to (a) 3^2, (b) $4^2 3^1$ and (c) 4^2 periodic states using the S^L notation.

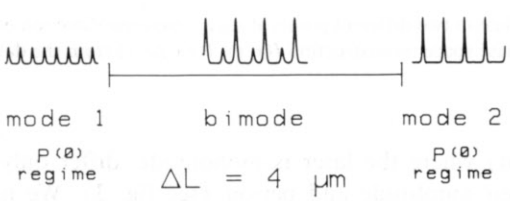

Fig. 3. Schematic representation of the range of bimode operation. Many other states are observed on each side of the locked state appearing in the middle of the range of bimode operation. 1 μm corresponds to a 10 MHz detuning for each mode.

it was done in ref. [16]. In our experiments, periodic states consist of repetitions of basic patterns with S small peaks followed by L large peaks. Such a pattern will be called a S^L state. With this notation, the $P^{(0)}$ monomode regimes that are obtained on each side of the bimode region are noted 1^0 (mode 1) and 0^1 (mode 2) respectively, while the bimode periodic regime occurring in the middle of the ΔL interval is of the 1^1 type as one period contains only a small peak followed by a large one. Some observed periodic states consist of two basic patterns that are formed from concatenation of nearby states and occur always between them. An example is given in fig. 4 that shows a part of the observed sequence. The various periodic signals have been classified using the Farey arithmetic where the sum of two rational numbers p_1/q_1 and p_2/q_2 is given by the ratio of the sum of their numerators over the sum of their denominators. This sum is the rational mediant between the two rational numbers with the smallest denominator. As in ref. [16], each state is labeled a firing number F defined by

$$F=\left(\sum L_i\right)\Big/\left(\sum (L_i+S_i)\right).$$

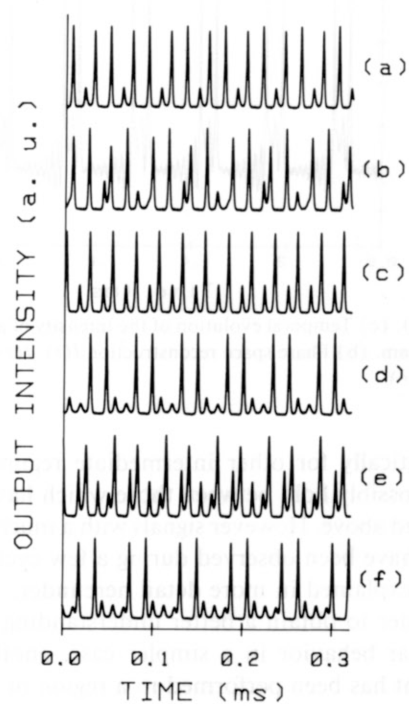

Fig. 4. Part of the sequence of periodic states obtained at increasing cavity length. Periodic states that are successively observed are, (a) 1^2, (b) $(1^2)^2$, (c) 1^1, (d) 2^2, (e) $1^1 2^1$, (f) 3^2 in the S^L notation.

This formula applies either to simple or concatenated states and assures F to be comprised between 0 and 1. Note that a state labeled F is not unique, an example is given by the $2^1 1^1$ and 3^2 states that are strongly different but have the same firing number (see fig. 4). Due to thermal frequency drifts in the laser, concatenated states with longer period or simple states with high values of S or L have been observed only during a few cycles. States with even longer period could probably exist between two periodic states but over a ΔL range far too narrow to be accessible with our experimental resolution. Many periodic regimes have been obtained in which the frequencies of the two passive Q-switching modes lock together in successive rational ratios that can be represented by a Farey tree. Fig. 5 shows such a tree in which the experimentally observed states have been reported using the firing number and states obtained via the Farey arithmetic have been connected by straight lines. Note that the sum rule applies here to the firing number defined above instead of the "winding number" as defined in periodically forced systems [22]. The relationship between the observed periodic sequence and the Farey arithmetic suggests that the bimode LSA, in spite of its complexity, may be simply viewed as a nonlinear system with two frequencies of self-pulsation. Erratic regimes that could be either quasiperiodic or possibly chaotic have also been recorded in various regions of the parameter space, they will be analyzed in a following paper.

The CO_2 LSA exhibits not only the temporal behaviors reported above but also transverse effects.

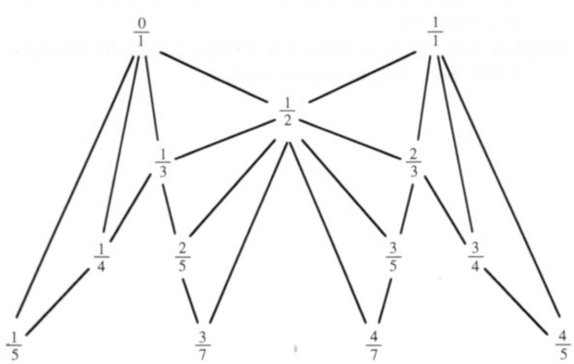

Fig. 5. Farey tree obtained by combinations of firing numbers in the two-mode LSA operating in $P^{(0)}$ regimes.

We have studied the evolution of the temporal signal along a transverse direction of the output beam. In addition to a trivial radial dependence of the steady state intensity, it has been observed that unlike the monomode case, the signals taken from two different points at distances r and $r+\Delta r$ from the axis of the laser are not proportional to each other. For example, fig. 1c represents the temporal evolution of the laser output in the same conditions as in fig. 2a, but at different r distances. Fig. 1a, c show that the ratio of the amplitudes of the two signals is not constant with time. This is particularly true during the time of dual emission when the system leaves the vicinity of one fixed point to approach the other one. To give a comparison of the instantaneous intensity sum evolution in two different points of the beam and to show evidence of spatial effects, the signals have been monitored simultaneously with two HgCdTe detectors. Fig. 1d is a 2D phase space reconstruction $I(r, t)$, $I(r+\Delta r, t)$ from two time series taken in two different points of the transverse profile. In monomode operation, the trajectory lies on a straight line even if I is not constant with time, i.e. when the laser is Q-switching. This means that the whole transverse profile is pulsating in an homogeneous way. Note that this reconstruction looks quite similar to fig. 1b except that the plane manifolds containing I_1 and I_2 respectively have not the same orientation as in fig. 1b. This reconstruction may be easily interpreted in considering that during the time when the LSA emits a monomode output intensity, i.e. in the spiraling part of the trajectory around the fixed points, there is no transverse effect, as it is the case in monomode regime. Fig. 1d shows that during this time, the intensities are proportional as the spiraling parts of the trajectory appear in a direction perpendicular to the plane of reconstruction. Spatial effects are prominent during the small time duration of dual emission, when the signals taken from two different points are not simply related. The shape of the trajectory in the $I(r, t)$, $I(r+\Delta r, t)$ phase space is a consequence of the dephasing between the two signals that depends on the Δr distance between the two detectors.

In this preliminary investigation of a bimode CO_2 LSA, a wide variety of phenomena has been obtained. It will be completed by the theoretical analysis of a model that could provide a realistic description of the coupling between modes. A complete treatment requires the use of a formalism like that of Oppo et al. [23] where the different dynamical laser variables (E, P, D) are developed on a basis of well suited eigenfunctions like the Gauss–Laguerre modes in the case of ref. [23]. As the correct treatment of the CO_2 laser with an intracavity absorber requires a three-level model for the amplifier and at least a two-level one for the absorber, the complete treatment is far too involved to allow analytical studies and requires heavy computing. Since the most important point is to understand what are the key ingredients responsible for the experimentally observed dynamics of the bimodal LSA, it is worth trying to set up a model as simple as possible but that would not exclude the basic mechanisms from which these dynamics originate.

In that purpose, we have developed the different dynamical variables of the LSA on the minimum basis for two-mode operation, i.e. including only two eigenfunctions and in which we have considered various coupling terms similar to those arising from the complete theory. In the numerical simulations undertaken on the basis of this model, mode hopping has been found as it appears experimentally and the temporal signals exhibit patterns in good qualitative agreement with the experiments.

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