Laser Chaotic Attractors in Crisis

Didier Dangoisse, Pierre Glorieux, and Daniel Hennequin
Laboratoire de Spectroscopie Hertzienne, Université de Lille I,
59655 Villeneuve d'Ascq Cedex, France
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Different crises, i.e., abrupt qualitative changes in the properties of attractors, have been observed in a CO_2 laser with internal modulation. They are shown to be related to crossing between a stable or unstable periodic cycle and the strange attractor resulting from the period-doubling cascade observed at low modulation levels. Depending on the operating conditions, boundary and interior crises have been observed.

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According to Grebogi, Ott, and Yorke,[1] a crisis is a sudden change of the behavior of an attractor. Such crises were observed in model electronic circuits by Hilborn[2] and by Rollins and Hunt.[3] We report here the first observation and characterization of different crises of the attractors in a laser, in the particular case of a CO_2 laser containing an amplitude or frequency modulator.

The CO_2 laser with modulated parameter has recently been shown to present a very clear cascade of period-doubling bifurcations culminating in chaos with periodic windows.[4,5] This system has many practical applications since it provides the most efficient way of getting modulated laser radiation for, e.g., telecommunication uses. From the point of view of the physicist it is very attractive for the study of nonlinear phenomena (i) because the laser allows a very large signal-to-noise ratio, (ii) because of the time scale of the effects, and (iii) because of its very rich phenomenology. For instance, this system displays "generalized bistability," i.e., bistability between different attractors,[5] dynamic deformations of the bifurcation diagram.[4,5] Moreover the dimension of the strange attractor has been measured to be close to that calculated with a simple model of the laser, which thus provides a powerful support for numerical description of lasers with internal modulation.[6]

Our experimental system is based on a sealed-off waveguide CO_2 laser with a grating for line selection. The pressure of the laser-gas mixture (He, 8; CO, 1; CO_2, 1) is typically 100 mbar, and thus the active medium is essentially homogeneously broadened. A ZnSe elasto-optic modulator is inserted in the laser cavity which is 30 cm long. The mode width is typically 200 MHz and the cavity free spectral range 500 MHz. Depending on the frequency or the modulating signal, different vibrational modes of the crystal are excited, resulting in AM, FM, or AM + FM modulation. In the present status of the laser, both cavity frequency and losses are modulated but as the loss modulation is more efficient, the contribution of the cavity FM may be neglected. The modulation frequency, 330 kHz, is close to that calculated for the relaxation oscillations of the laser (300 kHz). The intensity of the laser is monitored by a fast HgCdTe photovoltaic detector whose output is usually stored in a digital oscilloscope. The laser intensity may also be sampled at some definite time, at each period of the modulation voltage. The output of the sampler displayed versus the control parameter provides very clear visualizations of the bifurcation diagram (BD). Such diagrams have been obtained by use of the modulation amplitude or frequency or the laser cavity length as the control parameter, but the first one appears to be the best suited for this study. Typical BD's obtained in these conditions are given in Ref. 5. Phase diagrams in which the laser output I is displayed versus its time derivative $\dot{I}$ also proved useful in the investigation of crises in the laser attractor. In these representations, a crisis is revealed by the sudden appearance of new explored regions in the $(I,\dot{I})$ plane or in the BD.

Different crises have been observed as the cavity detuning is changed. First, when the laser is only slightly

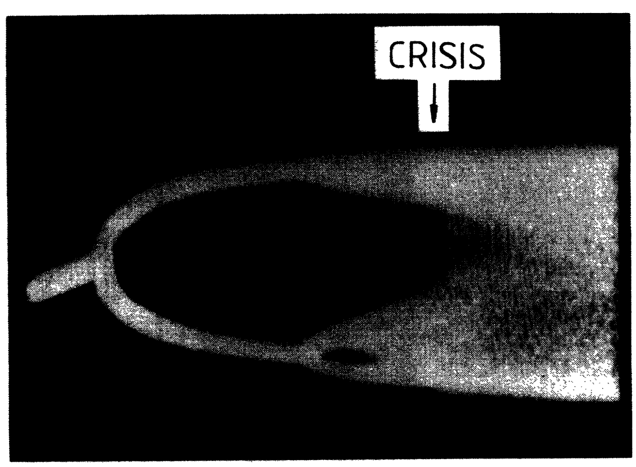

FIG. 1. Bifurcation diagram of the laser showing a period-doubling sequence to chaos and a crisis corresponding to an expansion between the two branches of the chaotic regime.

FIG. 2. Bifurcation diagram of the laser with a larger range of variation of the driving voltage. The crisis corresponds here to a sudden expansion of the chaotic attractor.

detuned from resonance, at some critical value a new branch appears in between the two branches previously visited by the attractor (see Fig. 1). This expansion does not destroy the attractor but only expands it. This behavior of an attractor was called interior crisis by Grebogi, Ott, and Yorke.[1]

Second, when the laser cavity is detuned by about 50 MHz, there is a domain in which generalized bistability occurs. Then another attractor may coexist with the chaotic attractor issued from the period-doubling cascade. That attractor may appear as a $3T$ cycle and exhibits period doubling and a chaotic regime when the control parameter is increased. Two different kinds of crises are observed depending upon whether the collision occurs in a region of periodic or chaotic regime of this second attractor.

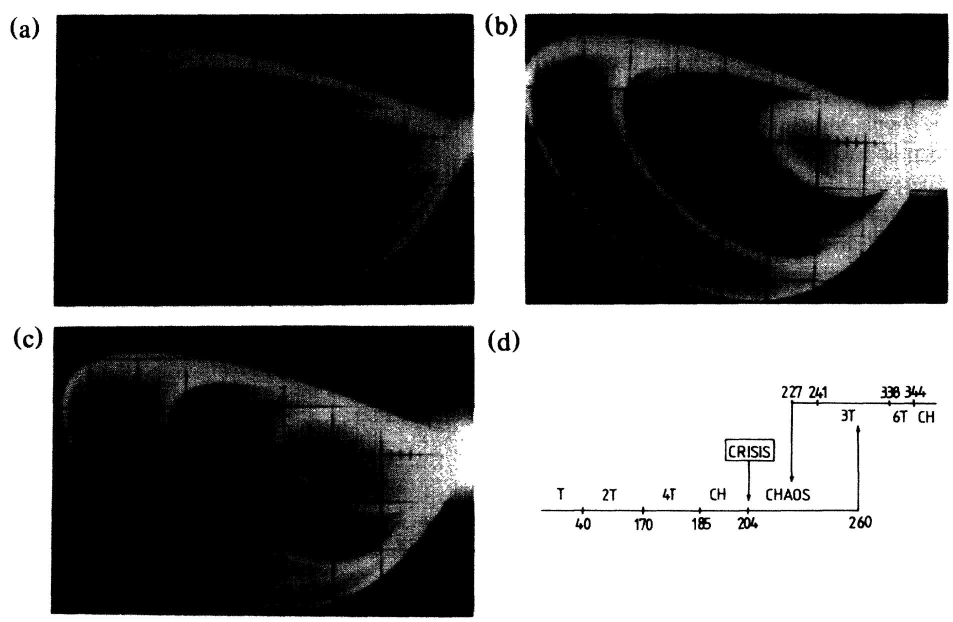

FIG. 3. $(I,\dot{I})$ plots in the crisis region at (a) 200 mV, (b) 204 mV, and (c) 208 mV driving voltages. The crisis is the same as that displayed in Fig. 1. (d) The situation of the crisis with respect to the bistability domain is indicated.

(a) When the collision occurs with the $3T$ stable cycle, a boundary crisis is produced when the control parameter V is increased above some critical value. Then the chaotic attractor disappears and a $3T$ cycle is observed. When V is further increased, the attractor evolves towards a $6T$ stable cycle and later to chaos. Such a behavior is visible, for instance, in Fig. 2 of Ref. 5(a).

(b) When the collision occurs with a chaotic part of the second attractor or in a region where the $3T$ cycle is unstable, the original chaotic attractor suddenly expands (Fig. 2). This sudden qualitative change of the attractor is not a complete destruction of the chaotic attractor since the statistics of the sampled value shows that there is some continuity between the attractors before and after the crisis. Thus this may be classified as an internal crisis.

Note that the $3T$ periodic regime is not the usual periodic window surging by a tangent bifurcation inside the chaotic domain. This statement is supported by three facts: (i) There are conditions in which generalized bistability between the chaotic attractor and the $3T$ cycle exists, (ii) the width of the region in which the $3T$ regime occurs is much broader than that associated with the usual $3T$ periodic window, and (iii) the sampled values in the $3T$ regime are often well outside the region visited by the chaotic attractor.

Figure 3 displays phase diagrams $(I,\dot{I})$ in the vicinity of an internal crisis similar to that shown in Fig. 1. The evolution of the laser attractor in the experimental conditions of these diagrams is summarized in the lower part of this figure. After the period-doubling cascade, the attractor becomes chaotic for a modulation amplitude $V_{\mathrm{mod}} = 185$ mV.[7] As V_{mod} is increased, the attractor evolves smoothly until it reaches the critical value $V_{\mathrm{crisis}} = 204$ mV where a new branch of the phase diagram suddenly appears in between the two main branches. As V_{mod} increases further, the laser attractor follows more and more frequently this branch which appears brighter in Fig. 3(c). Eventually this new branch and the two main branches before the crisis smear out together. When the modulation is increased further, the laser jumps to the $3T$ periodic regime in the boundary crisis discussed in (a) above and later on again to chaos. When the modulation amplitude is decreased, the $3T$-periodic-cycle region broadens and extends to a modulation range where the laser was previously chaotic. This is the generalized bistability effect[4,5(b)] and a crisis is also observed when V_{crisis} is reached from above. In Fig. 3(c) it is shown that the region in between the upper and lower chaotic branches is not evenly explored just after the crisis. This does not appear in Fig. 1 because that photograph was heavily overexposed to show more clearly the new (intermediate) branch.

The assignment of the observed changes of the BD's to internal crises due to the collision of the chaotic attractor

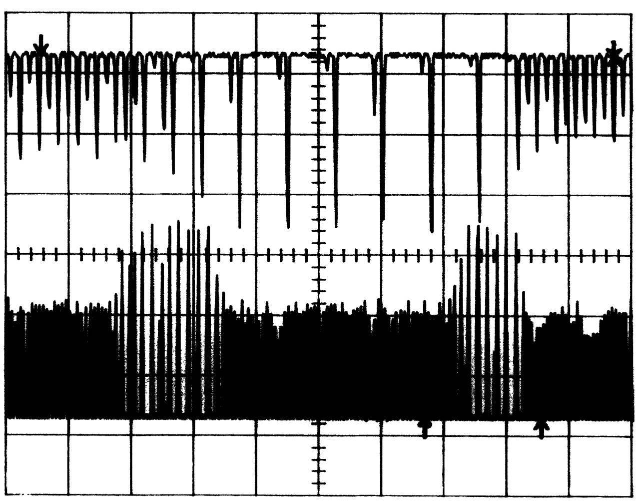

FIG. 4. Pseudoperiodic bursts in the intensity above the crisis. The upper trace is an expanded view of the part of the lower trace between arrows.

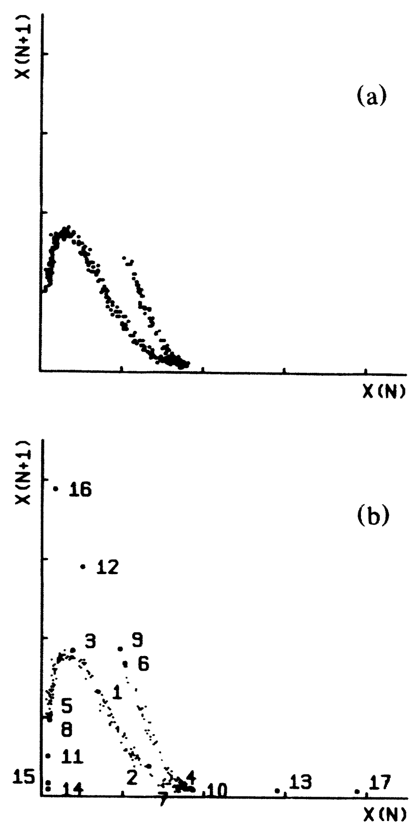

FIG. 5. Return maps of successive laser intensity maxima (I_n, I_{n+1}) (a) below and (b) above a crisis similar to that displayed in Fig. 2. The numbers indicate the chronology of the crisis and refer to the successive values of n. From points 3, 4, and 5, the system evolves on various limits of the "attractor." From point 9, it is well outside the attractor.

with unstable periodic cycles is supported by the direct observation of the time dependence of the laser intensity just after the crisis. This is particularly clear in the case of the crisis reported in Fig. 2 where the newly explored branches of the attractor correspond to intensity peaks much larger than in the original chaotic regime. Then, as shown in Fig. 4, this original chaotic regime is interrupted by bursts of (quasi)periodic pulsations. At some irregular times, the laser temporarily jumps into an unstable cycle regime until it is eventually trapped again in the chaotic attractor. In the conditions of Fig. 4 the two regimes (chaotic and unstable cycle) are easily distinguished because they correspond to spikes with quite different intensities.

To investigate more precisely the onset of a crisis, we have plotted another kind of return map which is associated with the crossing of the attractor with the plane $\dot{I}=0$. Time series of the output intensity were recorded and the amplitude of the nth maximum I_n has been plotted versus that of the next maximum I_{n+1}. For driving amplitudes V less than the crisis value, all (I_n,I_{n+1}) points remain located on a "curve" as shown in Fig. 5(a). When V is increased, this "curve" expands. As $V=V_c$, it reaches a point where the system escapes the attractor; it explores points well outside the region shown in Fig. 5(a). A typical scenario near the critical point is illustrated in Fig. 5(b) where the numbers refer to the chronology of the crisis. The experimental investigation of a number of crises of this kind indicate that the scenario is always the same for different bursts of nT-pseudoperiodic pulses and thus may be considered as a signature of the particular crisis under investigation.

In addition to the two kinds of crises presented in the first part of this paper, the laser with internal modulation presents another crisis when operated just above threshold. In these conditions, the transition to chaos does not appear through the period-doubling sequence but the T-periodic regime slowly evolves to a completely irregular behavior. Spikes in the laser intensity have amplitude fluctuations which become larger and larger as the control parameter V is increased. The corresponding bifurcation diagram indicates that above a critical value of V, the system explores new branches more and more frequently as V is further increased.

To conclude, different crises have been observed in a CO_2 laser with internal modulation. The different tools such as bifurcation diagrams and return maps allow us to characterize these crises. Most of them are due to the crossing of an unstable periodic cycle with the strange attractor associated with the chaotic regime. Other crises should be observed in other laser systems and this should still enlarge the variety of chaotic phenomena they present.

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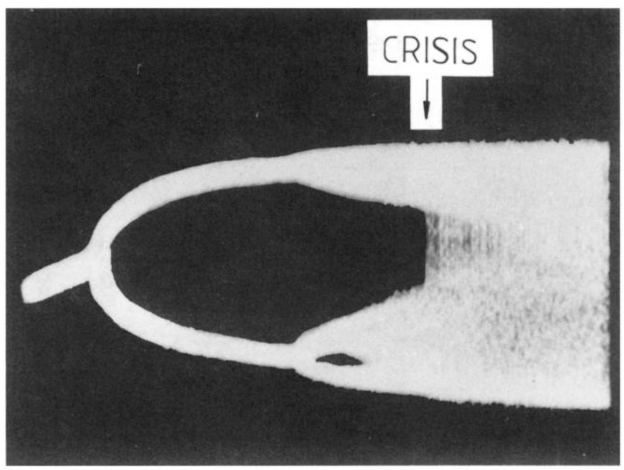
[7]A 100-mV peak-to-peak voltage at 330 kHz induces a loss inside the cavity of 0.5% peak to peak.



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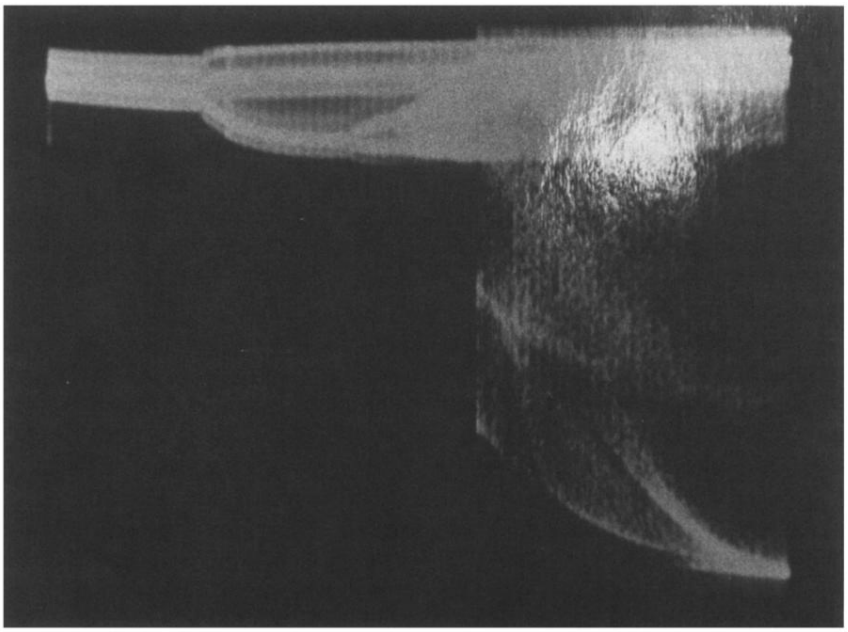

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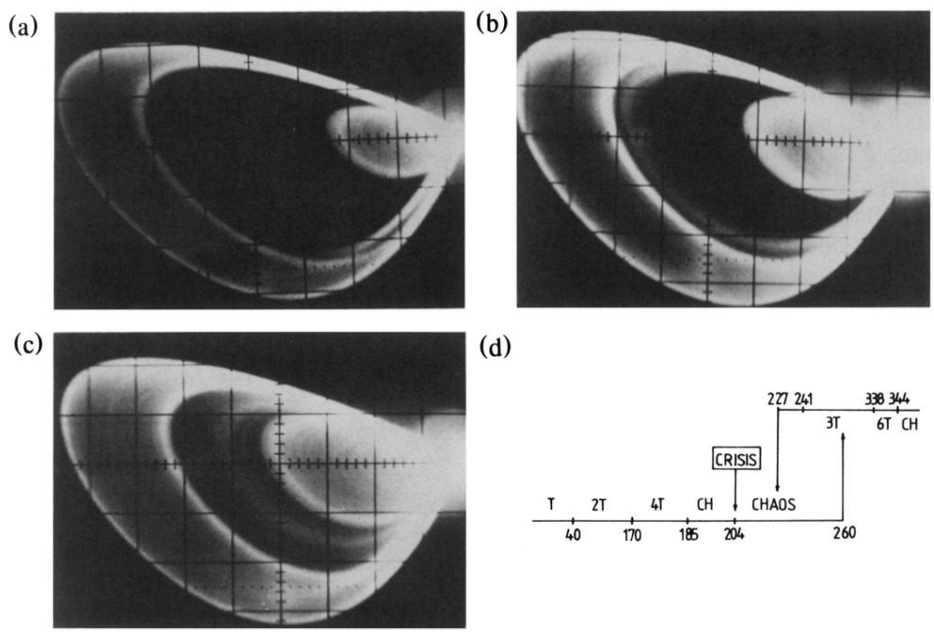

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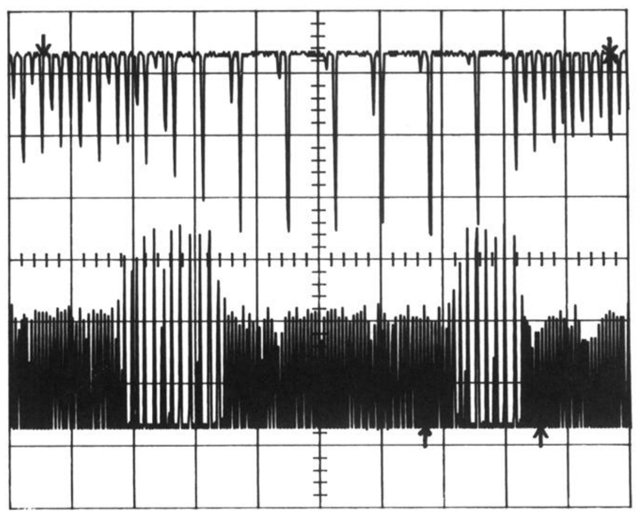

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